

RL Circuit and Time Constant

Object: To investigate the voltages across the resistor and inductor in a resistor-inductor circuit (RL circuit), and the current through the resistor and inductor so that the behavior of an inductor in a DC circuit can be studied. We also wish to determine the inductive time constant for the circuit.

Apparatus: Power Amplifier, multimeter, (2) voltage sensors, patch cords, resistor 10 ohm, inductor 8.2 milliHenry, computer, PASCO Interface and Data Studio Software.

FOREWORD

When a DC voltage is applied to an inductor and a resistor in series, a steady current will be established:

$$(1) \quad I_{\max} = \frac{V_o}{R}$$

where V_o is the applied voltage and R is the total resistance in the circuit. But it takes time to establish this steady-state current because the inductor creates a back emf in response to the changing current. The current at any time is given by:

$$(2) \quad I = I_{\max} (1 - e^{-(R/L)t}) = I_{\max} (1 - e^{-(t/\tau)})$$

where L is the inductance, R is the total resistance and the quantity $L/R = \tau$ is the inductive time constant. The inductive time constant is a measure of how long it takes the current to be established. One inductive time constant is the time it takes for the current to rise to 63% of its maximum value (or fall to 37% of its maximum). The time for the current to rise or fall to half its maximum is related to the inductive time constant by

$$(3) \quad \begin{aligned} t_{1/2} &= \tau (\ln 2) \\ \tau &= \frac{t_{1/2}}{\ln 2} \end{aligned}$$

Since the voltage across a resistor is given by $V_R = IR$, the voltage across the resistor is established exponentially:

$$(4) \quad V_R = V_o (1 - e^{-t/\tau})$$

Since the voltage across an inductor is given by

$$(5) \quad V_L = L \left(\frac{dI}{dt} \right)$$

the voltage across the inductor starts at its maximum and then decreases exponentially:

$$(6) \quad V_L = V_o e^{-(t/\tau)}$$

After a time $t \gg \tau$, a steady-state current I_{\max} is established, the voltage across the resistor is equal to the applied voltage, V_O and the voltage across the inductor is zero. If, after the maximum current is established, the voltage source is turned off, the current will then decrease exponentially to zero while the voltage across the resistor does the same and the inductor again produces a back emf that decreases exponentially to zero. In summary:

DC Voltage applied:

$$I = I_{\max} (1 - e^{-(t/\tau)})$$

$$V_R = V_O (1 - e^{-(t/\tau)})$$

$$V_L = V_O e^{-(t/\tau)}$$

$$V_R + V_L = V_{\text{emf}}$$

DC Voltage turned off:

$$I = I_{\max} e^{-(t/\tau)}$$

$$V_R = V_O e^{-(t/\tau)}$$

$$V_L = -V_O e^{-(t/\tau)}$$

$$V_R + V_L = 0$$

At any time, **Kirchhoff's Loop Rule** applies: The algebraic sum of all the voltages around the series circuit is zero. In other words, the voltage across the resistor plus the voltage across the inductor will add up to the source voltage.

PROCEDURE

In this activity, the **Power Amplifier** provides voltage for a circuit consisting of an inductor and a resistor in series. **Voltage Sensors** will be used to measure the voltage across the inductor and resistor. The **Signal Generator** produces a low frequency square wave that imitates a DC voltage being turned on and then turned off.

We will record and display the voltage across the inductor, the voltage across the resistor, and the output voltage of the power amplifier as the current is established exponentially. Note that the voltage across the resistor is in phase with the current. The voltage is also proportional to the current ($V=IR$). Therefore, the behavior of the current is studied indirectly by studying the behavior of the voltage across the resistor. We will use the graph display of these voltages to investigate the behavior of the inductor-resistor circuit and to determine the inductive time constant for the circuit.




Part I. Equipment and Software Set-up

Initial Data Collection


1. Use a multimeter to determine the resistance of the 10 Ohm resistor on the circuit board and record the value on the data sheet.
2. Use a multimeter to determine the resistance of the 8.2 mH Inductor on the circuit board and record the value on the data sheet.

Initial Equipment and Software Set-up




1. Start Data Studio, and select *Create Experiment*.
2. Connect the Power Amplifier to the computer via an analog port on the PASCO interface. Plug the Power Amplifier into an AC outlet, and turn the power amplifier on. Use a Red banana wire to connect the Power Amplifier to the 10Ω Resistor. Use a Black banana wire to connect the Power Amplifier to the 8.2 mH Inductor such that the resistor and inductor are in series.



3. Inform the software which analog port you plugged the Power Amplifier into by selecting the Power Amplifier icon and dragging it to the appropriate analog port.
4. A signal generator box should appear. Change the wave pattern from the default *Sine Wave* to a *Positive Square Wave*. Note that a *Positive Square Wave* and a *Square Wave* are two different types of wave forms. Since we are using the Signal generator as our switch, the wave form needs to be a *Positive Square Wave* so that the voltage will alternate between 0.00 and 3.00 volts. If a normal *Square Wave* is used, the voltage will alternate between -3.00 and 3.00 volts. Change the frequency to 50.00Hz. Change the amplitude to 3.00V. The **Auto** button should already be selected. It is set to **Auto** so that the signal generator will start and stop automatically when the **Start** button is pressed.
5. Click the  labeled *Measurements and Sample Rate*, and change the Sampling Rate to 5000 Hz.
6. Minimize the signal generator box.
7. Connect a voltage sensor across the 10 Ω resistor, and connect it to the computer via an analog port on the PASCO interface.
8. Inform the software which analog port you plugged the voltage sensor into by selecting the voltage sensor icon and dragging it to the appropriate analog port
9. Connect a voltage sensor across the 8.2 mH Inductor, and connect it to the computer via an analog port on the PASCO interface.
10. Inform the software which analog port you plugged the voltage sensor into by selecting the voltage sensor icon and dragging it to the appropriate analog port.
11. Click the **Start/Stop Options** button , and select the *Automatic Stop* tab. Select the time option and input 0.02 seconds. Click **OK**.
12. Create a graph of the Voltage Across the Resistor vs. time.
13. Double click somewhere within the body of the actual graph, or click on the **Graph Settings** button  located on the graph toolbar to open the Graph Settings window. Select the layout tab, and under Group Measurements select the *Do Not Group* option. Next select the Tools tab. Under the Smart Tools, set the *Data Point Gravity* to 0. Select **OK**.

Part II. Collection of Data and Analysis of the Time Constant



1. Click the **Start** button. The software will collect data for 0.02 seconds and automatically stop.
2. Turn off the Power Amplifier.
3. Click the **autoscale**  button located on the graph tool bar.
4. Maximize the graph to fill the screen.
5. Magnify a section of the graph where the voltage of the Resistor is increasing. Stretch the time scale if necessary

- Determine the amount of time it takes to reach one half of the maximum voltage using the smart tool.

Click on the **Smart Tool** button . When the smart tool cursor has the following appearance , it can be moved to any location on the graph. Place the smart cursor at the location where the voltage of the Resistor begins to rise. By moving the mouse slightly to the 2nd quadrant of the smart tool cursor, you should be able to change the cursor to the delta cursor . When the delta cursor is present, left click and drag the cursor to the point where the voltage is one half of the maximum. The difference in time between these points is the time it takes the voltage (or current) to reach half of its maximum. It is noted as $t_{1/2}$.

- Use this information to determine the inductive time constant. Compare this value to the value calculated from L and R (do not forget to use the total R – not just the resistor)
- Print the graph with the smart tool data visible.
- Click the **autoscale**  button on the graph.
- Click on the **smart tool** button  so that the smart tool is off.

Part III. Completion of Software Set-up and Data Analysis.

- Add a graph of the Voltage Across the Inductor vs. Time to the existing graph. Click and drag the icon that represents the Voltage Across the Inductor to the existing graph. When the entire graph is boxed in a dotted line, drop the icon.
- Repeat step one for the Output Voltage of the signal generator.
- Make sure that the **align x-axis lock** button  has been selected.
- Click on each graph one at a time and turn on the smart tool.
- Make the Voltage Across the Resistor vs. time graph active by clicking on it.
- Click the **autoscale**  button on the graph.
- Magnify a section of the graph in an area where the voltage of the Resistor is increasing.
- Pick a point where the voltage of the Resistor is increasing and determine the voltage of the Resistor using the smart tool. Also determine the voltage of the Inductor and Output Voltage for the same time. Print the graph with the smart tool data visible. Comment on the result.
- Repeat steps 5 and 8 for a point where the voltage of the Resistor is remaining stable. Print the graph with the smart tool data visible. Make sure you comment on the results.
- Repeat steps 5 and 8 for a point where the voltage of the Resistor is decreasing. Print the graph with the smart tool data visible. Make sure you comment on the results.

DATA AND CALCULATION SUMMARY

Inductor resistance $R_L = \underline{\hspace{2cm}} \Omega$

Resistor resistance $R_R = \underline{\hspace{2cm}} \Omega$

Total resistance $R_{\text{Total}} = R = \underline{\hspace{2cm}} \Omega$

Peak voltage (for resistor) $V = \underline{\hspace{2cm}} \text{ Volts}$

Time when voltage across the resistor starts to increase $T_{\text{start}} = \underline{\hspace{2cm}} \text{ sec}$

Time at half-maximum voltage across the resistor $T_{\text{Half Max}} = \underline{\hspace{2cm}} \text{ sec.}$

Time for the voltage to rise to half of its maximum value $= t_{1/2} = T_{\text{Half Max}} - T_{\text{start}} = \underline{\hspace{2cm}} \text{ sec}$

Now determine the time constant by the following two methods:

$$\tau = \frac{t_{1/2}}{\ln 2} \underline{\hspace{2cm}} \text{ sec} \quad = L/R \underline{\hspace{2cm}} \text{ sec}$$

Voltages and comments for a time when the current is increasing:

Voltages and comments for a time when the current is stable:

Voltages and comments for a time when the current is decreasing:

